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CP PUBLICATION

SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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SHORT TRICKS
Mathematics
for JEE (Main & Advanced)

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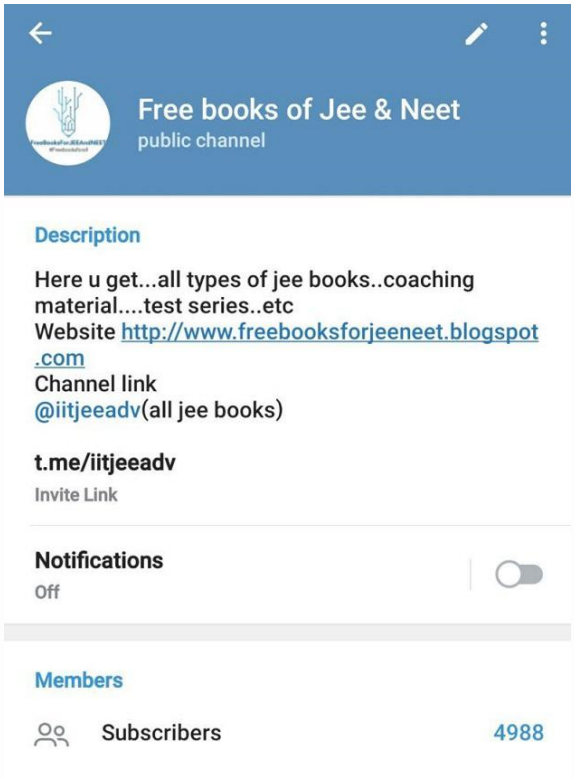

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Limit

KEY CONCEPTS

1. Indeterminate Form

$0 \times \infty$, 0^0, 1^∞, $\infty - \infty$, ∞/∞, ∞^0, $0/0$.

2. Properties of Logarithms

Let M and N arbitrary positive number such that $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ then

(i) $\log_a MN = \log_a M + \log_a N$

(ii) $\log_a \frac{M}{N} = \log_a M - \log_a N$

(iii) $\log_a N^\alpha = \alpha \log_a N$ (α any real no.)

(iv) $\log_{a^\beta} N^\alpha = \frac{\alpha}{\beta} \log_a N \quad (\alpha \neq 0, \beta \neq 0)$

(v) $\log_a N = \frac{\log_b N}{\log_b a}$

(vi) $\log_b a . \log_a b = 1 \Rightarrow \log_b a = \frac{1}{\log_a b}$

(vii) $e^{(\ln a^x)} = a^x$

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3. Logarithmic Inequality

Let a is real number such that

(i) For $a > 1$ the inequality $\log_a x > \log_a y$ & $x > y$ are equivalent.

(ii) If $a > 1$ then $\log_a x < \alpha \Rightarrow 0 < x < a^\alpha$

(iii) If $a > 1$ then $\log_a x > \alpha \Rightarrow x > a^\alpha$

(iv) For $0 < a < 1$ the inequality $0 < x < y$ & $\log_a x > \log_a y$ are equivalent

(v) If $0 < a < 1$ then $\log_a x < \alpha \Rightarrow x > a^\alpha$

4. Important Discussion

(i) Given a number N, Logarithms can be expressed as

$\log_{10} N$ = integer + fraction (+ve)

Characteristics Mantissa

(a) The mantissa part of log of a number is always kept positive.

(b) if the characteristics of $\log_{10} N$ be n then the number of digits in N is (n + 1)

(c) If the characteristics of $\log_{10} N$ be (–n) then there exists (n –1) number of zeros after decimal point of N.

(ii) If the no. & the base are on the same side of the unity, then the logarithm is positive; and If the no. and the base are on different side of unity. Then the logarithm is negative.

5. Limits of a Function

$\lim_{x \to a} f(x) = \ell$

For finding right hand limit of the function we write (x + h) in place of x while for left hand limit we write (x – h) in place of x.

6. Existence of Limit

Let f be a function in "x". If for every positive number ϵ, how ever small it may be there, $\exists$ a positive number δ such that whenever $0 < |x - a| < \delta$ we have $|f(x) - \ell| < \epsilon$ then we say, f(x) tends to limit "ℓ" as x tends to "a" and we say $\lim_{x \to a} f(x) = \ell$

Also in then neighbourhood of "a"
Right hand limit = Left hand limit
or $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$
Then we say limit of function " f " exist as $x \to a$
How to find R.H.L. and L.H.L
For R.H.L. :→ Replace x by "a + h" and write R.H.L. = $\lim_{h \to 0} f(a + h)$

For L.H.L.:→ Replace x by "a – h" and write L.H.L. = $\lim_{h \to 0} f(a - h)$

Also R.H.L. can be written as $\lim_{x \to a^+} f(x)$ or f(a + 0)

and L.H.L. can be written as $\lim_{x \to a^-} f(x)$ or f(a – 0)

7. Methods of Evaluation of Limits

(i) When $x \to \infty$
In this case expression should be expressed as a function 1/x and then after removing indeterminant form, (If it is there) replace 1/x by 0.

(ii) Factorisation method
If f(x) is of the form $\frac{g(x)}{h(x)}$ and of indeterminate form then this form is removed by factorising g(x) and h(x) and cancel the common factors, then put the value of x.

(iii) Rationalisation method
In this method we rationalise the factor containing the square root and simplify and we put the value of x.

(iv) By using some standard expansion

(a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.$

(b) $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots.$

(c) $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots.$

(d) $\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots.$

(e) $a^x = 1 + (x \log a) + \frac{(x \log a)^2}{2!} + \frac{(x \log a)^3}{3!} + \ldots..$

(f) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots.$

(g) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots.$

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(h) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots$

(i) $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$

(j) $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

(k) $\tanh x = x - \frac{x^3}{3} + 2x^5 - \ldots$

(l) $\sin^{-1}x = x + \frac{x^3}{3!} + \frac{9x^5}{5!} + \ldots$

(m) $\cos^{-1}x = \frac{\pi}{2} - \left(x + \frac{x^3}{3!} + \frac{9x^5}{5!} + \ldots\right)$

(n) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$

(o) $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots.$

(p) $(1 + x)^{1/x} = e[1 - \frac{x}{2} + \frac{11}{24}x^2 - \ldots\ldots\ldots\ldots]$

(v) 'L' Hospital rule

If $\lim_{x\to a}\frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

(vi) By using some standard limits

(a) $\lim_{x\to0}\frac{\sin x}{x} = \lim_{x\to0}\frac{x}{\sin x} = 1;\ \lim_{x\to0}\sin x = 0$

(b) $\lim_{x\to0}\cos x = \lim_{x\to0}\left(\frac{1}{\cos x}\right) = 1$

(c) $\lim_{x\to0}\frac{\tan x}{x} = \lim_{x\to0}\frac{x}{\tan x} = 1;\ \lim_{x\to0}\tan x = 0$

(d) $\lim_{x\to0}\frac{\sin^{-1} x}{x} = \lim_{x\to0}\frac{x}{\sin^{-1} x} = 1$

(e) $\lim_{x\to0}\frac{\tan^{-1} x}{x} = \lim_{x\to0}\frac{x}{\tan^{-1} x} = 1$

(f) $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = \lim_{x\to0}(1 + ax)^{1/x} = e^a$

(g) $\lim_{x\to0}\frac{a^x - 1}{x} = \log_e a\ (a > 0)$

(h) $\lim_{x\to0}\frac{e^x - 1}{x} = 1$

(i) $\lim_{x\to a}\frac{x^n - a^n}{x - a} = n\,a^{n-1}$

(j) $\lim_{x\to0}\frac{\log(1+x)}{x} = 1$

(k) $\lim_{x\to0}\frac{(1+x)^n - 1}{x} = n$

(l) $\lim_{x\to\infty} \frac{\sin x}{x} = \lim_{x\to\infty} \frac{\cos x}{x} = 0$

(m) $\lim_{x\to\infty} \frac{\sin(1/x)}{(1/x)} = 1$

8. Theorems on Limits

The following theorems are very helpful for evaluation of limits-

(i) $\lim_{x\to a} [k\, f(x)] = k \lim_{x\to a} f(x)$, where k is a constant

(ii) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(iii) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(iv) $\lim_{x\to a} [f(x).g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

(v) $\lim_{x\to a} [f(x)/g(x)] = [\lim_{x\to a} f(x)]/[\lim_{x\to a} g(x)]$ provided $\lim_{x\to a} g(x) \neq 0$

(vi) $\lim_{x\to a} f[g(x)] = f[\lim_{x\to a} g(x)]$

(vii) $\lim_{x\to a} [f(x) + k] = \lim_{x\to a} f(x) + k$ where k is a constant

(viii) $\lim_{x\to a} \log\{f(x)\} = \log\{\lim_{x\to a} f(x)\}$

(a) The Logarithm of a number is unique i.e. No number can have two different log to a given base.

(b) From the definition of the Logarithm of the number to a given base 'a'.

$a^{\log_a N} = N$, $a > 0, a \neq 1$, & $N > 0$ is known as the fundamental logarithmic identity.

(c) $\log_e a = \log_{10} a \cdot \log_e 10$ or $\log_{10} a = \frac{\log_e a}{\log_e 10} = 0.434 \log_e a$.

(ix) $\lim_{x\to a} [f(x)]^{g(x)} = \{\lim_{x\to a} f(x)\}^{\lim_{x\to a} g(x)}$

(x) $\lim_{x\to\pm\infty} f(x) = \lim_{x\to 0} f(1/x)$

9. Some Limits which do not exist

(i) $\lim_{x\to 0} \left(\frac{1}{x}\right)$

(ii) $\lim_{x\to 0} x^{1/x}$

(iii) $\lim_{x\to 0} \frac{|x|}{x}$

(iv) $\lim_{x\to a} \frac{|x-a|}{x-a}$

(v) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$

(vi) $\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$

(vii) $\lim_{x\to 0} e^{1/x}$

(viii) $\lim_{x\to\infty} \sin x$

(ix) $\lim_{x\to\infty} \cos x$

(x) If function takes any of the following form, $\frac{0}{0}, \frac{\infty}{\infty}$, then L'HOSPITAL'S RULE is applied

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Note :

- L'HOSPITAL'S RULE can be repeated required number of times in same Question.

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SHORT TRICKS

Trick -1 $\lim_{n\to\infty}\frac{1^p+2^p+3^p+....+n^p}{n^q}=\frac{1}{q}$ (p, q ∈ N)
(when q – p = 1)

Q.1 $\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+.....+\frac{n}{1-n^2}\right]$ is equal to-

(A) 1 (B) 2 (C) –1/2 (D) 1/2

Sol. **[C]**

Proper Method

$\lim_{n\to\infty}\frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}+.....+\frac{n}{1-n^2}$

$\lim_{n\to\infty}\frac{1}{1-n^2}[1+2+.....+n]$

$\lim_{n\to\infty}\frac{1}{1-n^2}\times\frac{n(n+1)}{2}=\lim_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)}{n^2\left[\frac{1}{n^2}-1\right]}\times\frac{1}{2}=-\frac{1}{2}$

Short Trick

$=-\frac{1}{2}$ $(\because q-p=1)$

Q.2 The value of $\lim_{n\to\infty}\left(\frac{1}{1-n^4}+\frac{8}{1-n^4}+.....+\frac{n^3}{1-n^4}\right)$ is -

(A) 1 (B) 0 (C) –1/4 (D) None of these

Sol. **[C]**

Proper Method

$\lim_{n\to\infty}\frac{1}{1-n^4}+\frac{8}{1-n^4}+.....+\frac{n^3}{1-n^4}$

$=\lim_{n\to\infty}\frac{1}{1-n^4}[1^3+2^3+.....+n^3]$

$=\lim_{n\to\infty}\frac{1}{1-n^4}\left[\frac{n(n+1)}{2}\right]^2$

$=\lim_{n\to\infty}\frac{n^4\left[\left(1+\frac{1}{n}\right)^2\right]}{n^4\left[\frac{1}{n^4}-1\right]\times 4}=-\frac{1}{4}$

Short Trick

$=-\frac{1}{4}$ $(\because q-p=1)$

Q.3 $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+.....+\frac{n}{n^2}\right)$ equals-

(A) 0 (B) 1/2 (C) 2n (D) 2^n

Sol. **[B]**

Proper Method	Short Trick
$\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+.....+\frac{n}{n^2}\right)$ $=\lim_{n\to\infty}\frac{n(n+1)}{2n^2}=\lim_{n\to\infty}\frac{n^2\left[1+\frac{1}{n}\right]}{2.n^2}$ $=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{2}=\frac{1}{2}$	$=\frac{1}{2}$ $(\because q-p=1)$

Trick -2 Sum of infinite series of type

$$\lim_{n\to\infty}\left[\frac{1}{a_1b_1}+\frac{1}{a_2b_2}+\frac{1}{a_3b_3}+.....\right]=\frac{1}{a_1(b_1-a_1)}$$

Where a_1, a_2, a_3 ..... are in A.P. and b_1, b_2, b_3 ..... are in A.P and $b_1-a_1=b_2-a_2=$ .....

Q.4 $\lim_{n\to\infty}\left[\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n(n+1)}\right]$ equals-

(A) 1 (B) 0 (C) 1/2 (D) 2

Sol. **[C]**

Proper Method	Short Trick
$\lim_{n\to\infty}\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n(n+1)}$ $=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+....+\left(\frac{1}{n}-\frac{1}{n+1}\right)$ $=\lim_{n\to\infty}\frac{1}{2}-\frac{1}{n+1}$ $=\frac{1}{2}-0=\frac{1}{2}$	$=\frac{1}{(2)(1)}=\frac{1}{2}$

Q.5 $\lim_{n\to\infty}\left[\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+.....+\frac{1}{(4n-1)(4n+3)}\right]$

(A) 1/12 (B) 2/7 (C) 5/8 (D) 1/11

Sol. **[A]**

Proper Method	Short Trick
$\lim_{n\to\infty}\frac{1}{4}\left[\frac{(7-3)}{3.7}+\frac{(11-7)}{7.11}+\frac{(15-11)}{11.15}+....+\frac{(4n+3)-(4n-1)}{(4n-1)(4n+3)}\right]$ $\lim_{n\to\infty}\frac{1}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+....+\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right)\right]$ $\lim_{n\to\infty}\frac{1}{4}\left[\frac{1}{3}-\frac{1}{4n+3}\right]=\lim_{n\to\infty}\frac{1}{4}\left[\frac{1}{3}-0\right]$ $=\frac{1}{12}$	$=\frac{1}{(3)(4)}=\frac{1}{12}$

Trick -3 $\lim\limits_{x\to 0}\left(\dfrac{a_1^x+a_2^x+a_3^x+a_n^x}{n}\right)^{\frac{1}{x}} = (a_1 a_2 a_3 \ldots a_n)^{\frac{1}{n}}$

Q.6 $\lim\limits_{x\to 0}\left(\dfrac{1^x+2^x+3^x+\ldots\ldots+n^x}{n}\right)^{1/x}$ is equal to -

(A) $(n!)^n$ (B) $(n!)^{1/n}$ (C) $n!$ (D) $\ell n(n!)$

Sol. **[B]**

Proper Method	**Short Trick**
$\lim\limits_{x\to 0}\left(\dfrac{1^x+2^x+3^x+\ldots\ldots+n^x}{n}\right)^{1/x}$ $(1^\infty$ form$)$ $= e^{\lim\limits_{x\to 0}\left[\frac{1^x+2^x+3^x+\ldots+n^x}{n}-1\right]\times\frac{1}{x}}$ $= e^{\lim\limits_{x\to 0}\frac{1^x+2^x+3^x+\ldots\ldots+n^x-n}{nx}}$ $\left(\dfrac{0}{0}\text{form}\right)$ $= e^{\lim\limits_{x\to 0}\frac{1^x\log 1+2^x\log 2+3^x\log 3+\ldots+n^x\log x-0}{n}}$ $= e^{\frac{\log 2+\log 3+\log 4+\ldots\ldots+\log n}{n}} = e^{\frac{1}{x}\log(2\times 3\times\ldots\ldots\times n)}$ $= e^{\frac{1}{x}\log n!} = e^{\log(n!)^{1/n}} = (n!)^{1/n}$	$= (1.\ 2.\ 3.\ \ldots.n)^{1/n}$ $= (n!)^{1/n}$

Q.7 $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$

(A) $(abc)^{\frac{1}{3}}$ (B) $(abc)^{\frac{1}{2}}$ (C) $(abc)^{\frac{1}{6}}$ (D) $(4abc)^{\frac{2}{3}}$

Sol. **[A]**

Proper Method	**Short Trick**
$\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$ $(1^\infty$ form$)$ $= e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{a^x+b^x+c^x}{3}-1\right)}$ $= e^{\lim\limits_{x\to 0}\left(\frac{a^x+b^x+c^x-3}{3x}\right)}$ $\left(\dfrac{0}{0}\text{ form}\right)$ $= e^{\lim\limits_{x\to 0}\left(\frac{a^x\log_e a+b^x\log_e b+c^x\log_e c}{3}\right)}$ $= e^{\frac{1}{3}(\log_e a+\log_e b+c^x\log_e c)}$ $= e^{\log_e(abc)^{\frac{1}{3}}}$ $= (abc)^{\frac{1}{3}}$	$= (a.b.c)^{\frac{1}{3}}$

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Trick -4 **Sandwitch Thorem:** If $f(x) \le g(x) \le h(x)$, then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x) \le \lim_{x\to a} h(x)$

Q.8 $\lim_{n\to\infty} \frac{[x]+[2x]+[3x]+.....+[nx]}{n^2}$ is equal ([·] = G.I.F)

(A) x/2 (B) 2x (C) x/3 (D) x/4

Sol. **[A]**

Proper Method

for any integer we have

$kx - 1 < [kx] \le kx$

$$\Rightarrow \sum_{k=1}^{n}(kx-1) < \sum_{k=1}^{n}[kx] \le \sum_{k=1}^{n} kx$$

$$\Rightarrow \frac{1}{n^2}\sum_{k=1}^{n}(kx-1) < \frac{1}{n^2}\sum_{k=1}^{n}[kx] \le \frac{1}{n^2}\sum_{k=1}^{n}(kx)$$

$$\Rightarrow \frac{1}{n^2}\left(\frac{n(n+1)}{2}\right)x - \frac{n}{n^2} < \frac{1}{n^2}\sum_{k=1}^{n}[kx] \le \frac{1}{n^2}\frac{n(n+1)}{2}.x$$

$$\Rightarrow \left(\frac{n+1}{2n}\right)x - \frac{1}{n} < \frac{1}{n^2}\sum_{k-1}^{n}[kx] \le \left(\frac{n+1}{2n}\right)x$$

Talking limit as $n \to \infty$, we obtain

$$\frac{x}{2} - 0 < \lim_{n\to\infty}\frac{1}{n^2}\sum_{k-1}^{n}[kx] \le \frac{x}{2}$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n^2}\sum_{k-1}^{n}[kx] = \frac{x}{2}$$

Short Trick

Assume greatest integer function as a simple function

$$= \lim_{n\to\infty}\frac{x+2x+3x+....nx}{n^2}$$

$$= \lim_{n\to\infty}\frac{(1+2+....+n)x}{n^2}$$

$$= \frac{x}{2}$$

Q.9 If [x] denotes the greatest integer $\le x$, then $\lim_{n\to\infty}\frac{1}{n^3}\{[1^2x] + [2^2x] + [3^3x] + ... + [n^2x]\}$ equals ([·] = G.I.F) -

(A) x/2 (B) x/3 (C) x/6 (D) 0

Sol. **[B]**

Proper Method

$$\lim_{n\to\infty}\frac{[1^2x]+[2^2x]+[3^2x]+.....+[n^2x]}{n^3}$$

$$= \lim_{n\to\infty}\frac{1^2x-\{1^2x\}+2^2x-\{2^2x\}+....+n^2x-\{n^2x\}}{n^3}$$

$$= \lim_{n\to\infty}\frac{(1^2+2^2+....+n^2)x}{n^3} - \frac{(\{1^2x\}+\{2^2x\}+.....+\{n^2x\})}{n^3}$$

$$= \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}x - 0$$

$$= \lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\times x$$

$$= \frac{2}{6}\times x = \frac{x}{3}$$

Short Trick

Assume greatest integer function as a simple function

$$= \lim_{n\to\infty}\frac{1^2x+2^2x+3^2x+.....+n^2x}{n^3}$$

$$= \lim_{n\to\infty}\frac{(1^2+2^2+.....+n^2)x}{n^3}$$

$$= \frac{x}{3}$$